

Vanishing Point and Dilations

STEAM Lesson Grade 8

OBJECTIVES

MATH

- SWBAT describe the effect of dilations on a plane using coordinates.
- SWBAT understand that a dilation produces similar figures.
- SWBAT understand how mathematics is used in one-point perspective drawings/paintings.
- SWBAT find the scale factor of a dilation.

ELA

- SWBAT conduct a short research project on the history of zero and how it relates to art.
- SWBAT read and analyze informational texts and the interactions between ideas and events. SWBAT determine the meaning of words and phrases as they are used in informational texts.
- SWBAT recognize an author's perspective and purpose in a text and/or distinguish among facts, judgements and speculations in a text.

VISUAL ARTS

- SWBAT create a painting/drawing using vanishing lines and one-point perspective.
- SWBAT describe how social and cultural events affected art and vice versa.
- SWBAT to analyze art influences beliefs and values.

MATERIALS

- Access to internet
- Examples of one-point perspective paintings/drawing with an obvious vanishing point.
- Transparencies or sheet covers with dry erase fine tip markers
- Student Worksheet
- Grid Paper
- Art Supplies

STANDARDS

MATH

MATH PRACTICE STANDARDS

- **S.MP.2** Reason abstractly and quantitatively.
- **S.MP.3** Construct viable arguments and critique the reasoning of others.
- **S.MP.7** Look for and make use of structure.

ACADEMIC STANDARDS

- **8.G.3.** Describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates.
- **8.G.4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given

two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)

ENGLISH LANGUAGE ARTS

INFORMATIONAL TEXTS

- **RI.8.2** Analyze informational text development.
 - a. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas.
 - b. Incorporate central ideas and their relationships into an objective summary of the text.
- **RI.8.3** Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories).
- **RI.8.4** Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
- **RI.8.6** Determine an author’s perspective or purpose in a text and analyze how the author acknowledges and responds to conflicting evidence or viewpoints.
- **RI.8.10** By the end of the year, read and comprehend literary nonfiction at the high end of the grades 6–8 text complexity band independently and proficiently.

SCIENCE AND TECHNICAL SUBJECTS

- **RST.6-8.2** Analyze content-area-specific text development. a. Determine the central ideas or conclusions of a text. b. Provide an accurate and objective summary that includes the central ideas or conclusions of the text.
- **RST.6-8.4** Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6–8 texts and topics.
- **RST.6-8.8** Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.
- **RST.6-8.10** By the end of grade 8, read, comprehend, and respond to science/technical texts in the grades 6–8 text complexity band independently and proficiently.

WRITING

- **W.8.4** Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
- **W.8.7** Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.
- **W.8.9** Draw evidence from literary or informational texts to support analysis, reflection, and research.
 - b. Apply grade 8 Reading standards to literary nonfiction (e.g., “Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and

the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced”).

- **W.8.8** Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others, while avoiding plagiarism and following a standard format for citation.

FINE ARTS: VISUAL ARTS

- **8.1RE** Build relevant vocabulary to describe and analyze works of art.
- **8.4RE** Understand how cultural factors affect what contemporary artists create.
- **8.1CO** Interpret how community context, beliefs, and resources influence works of art.

DRIVING QUESTIONS

- How did the invention of zero affect art?
- What is the 0th dimension?
- How can the concept of the vanishing point help you understand math (similarity, scale drawings, dilations etc.)?

LESSON

INTRODUCTION

Question: *How do we double the size of a wiggly curve?*

Introduce this question before introducing One-Point perspective or doing any research about the 0th dimension. After a short discussion, set the question aside and continue steps A-C.

PART A: BUILDING BACKGROUND

HISTORY OF ZERO

Note: I recommend collaborating with the ELA teacher for this section of the lesson.

Although *Zero: The Biography of a Dangerous Idea* by Charles Seife presents an exciting history of Zero it's too long for a cross-curricular lesson for middle schoolers. Instead have your students use the internet to research the history of zero drawing on several sources. Make sure they know how to assess the credibility and accuracy of each source. When quoting or paraphrasing websites make sure your students cite their sources. Depending on time you can have each group research one or more of the research questions. After their writing assignment is complete, have them present their findings to the class.

Here are some questions for groups to research:

- Why was the invention of zero controversial?
- How has the invention of zero impacted art?

- What has the invention of zero allowed us to accomplish as a society?
- What are some challenges with the invention of zero?

There are many great websites out there. Here are a few that I found that might be helpful:

To align to the ELA standards have students analyze different websites and make connections between individual, ideas, and events that relate to zero. Have each group write down 10 words that they want to add to the class's vocabulary list. Discuss how author(s) respond to conflicting viewpoints about zero.

Homeschool Adaptation: Homeschoolers will need to research all four sections. If time is an issue, have them write shorter paragraphs instead of requiring a short research project.

PART B: MAKING CONNECTIONS

WHAT IS A DIMENSION?

Start the discussion by asking students what's the difference between a two-dimensional object and a three-dimensional object. Most middle schoolers can give examples of two and three dimensional objects. Then ask them "What's a one-dimensional object?" They might struggle a bit here. Then ask them "How about a zero-dimensional object?"

Once you've established that a mathematical point has zero dimensions (compared to a point that you make with a pencil or marker that has an actual diameter), discuss how many points there are on plane. Also, use this opportunity to discuss that a line has an infinite number of points, yet not all points on the plane are on a specific line. Note: Infinite \neq All

Tip: The penny illustration from *Flatland* by Edwin Abbott is great!

CONNECTION TO UNITS

Many students seem to struggle with the dimensionality of units. This is a great time to reinforce the topic. Discuss that since lines are one dimensional, we use units such as centimeters or inches. Since area is the amount of squares that it takes to cover a plane, we measure area with square units such as square centimeters (cm^2) or square inches (in^2). To measure volume, we quantify how many cubes fill up a space, so we measure volume in units such as cubic centimeters (cm^3) or cubic inches (in^3).

IMPACT OF ZERO ON ART: VANISHING POINT

Show students examples of art before the invention of the vanishing point and afterwards. Ask them what they noticed about the differences between the two types of paintings. Discuss how the invention of an infinite zero allowed Phillip Brunelleschi to invent the vanishing point.

Here is a video you might want to show:

[Why paintings used to look so weird, and how linear perspective changed everything | CBC Arts](#)

PART C: EXPLORING VANISHING POINTS

Note: You may want to collaborate with a fine arts teacher for Part C.

Display a picture on the board that has an obvious one-point perspective. Show students how to find the vanishing point. Give students some copies of photos or paintings with an obvious single vanishing point. Give a brief synopsis of the painter and the time frame/era that he/she lived in to build their cultural understanding of art.

Tip: If you want to use colored copies that you can laminate and reuse every year, give students a transparency sheet or clear plastic slip covers and overlay it on the painting to find the vanishing point.

Differentiated Learning Tip: For gifted students let them find the vanishing points in a painting with multiple vanishing points.

PART D: CONNECTING MATHEMATICS AND ART

SIMILARITY

Grade 7

In order to understand similarity students need to move from thinking additively to thinking multiplicatively. Two figures are similar if there is the same constant ratio applied to all its dimensions uniformly (no stretching). This means that similar figures have a proportional relationship both within and between corresponding sides. The proportional relationship *between* figures is called the **scale factor**.

Grade 8 students can build on students work with the constant of proportionality to make connections to the vanishing point and similar figures. See [Ohio's Grade 7 Math Model Curriculum with Instructional Supports](#) pages 100-102 for tips on how to teach similarity. A more expansive lesson on similarity and vanishing points for Grade 7 can be found on the Word & Numbers website.

High School Geometry

Whereas middle schoolers typically view similarity as two figures whose corresponding sides are proportional and angles are congruent, high school students view similarity in terms of transformations. From a transformational perspective two figures are similar if there is a series of transformations including a dilation that maps one figure onto the other. Specifically, two figures are similar if one figure is congruent to the dilation of the other. Therefore using one-point perspective and a vanishing point are helpful in giving a practical application to similarity and dilations. See [Ohio's High School Geometry Model Curriculum with Instructional Supports](#) pages 79- 98 for further information about dilations in high school.

Internal Comparison Within Figures



Scale Factor Between Figures

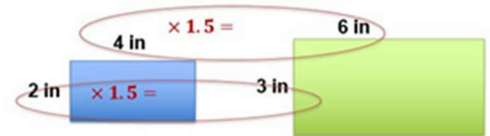


Image from the Ohio Department of Education's [7th Grade Model Curriculum for Mathematics with Instructional Supports](#).

SCALE FACTOR

The constant of proportionality, k , is a structural element that appears in a variety of contexts. Not only is k the slope in a linear graph that goes through the origin ($y = kx$), but it is also the scale factor in similar images.

The vanishing point is a great context to teach students about similarity and scale factor. Scale factor is the ratio of corresponding lengths in an image to the corresponding lengths in the preimage. For a full list of contexts and tips on how to teach scale factor see [Ohio's Grade 7 Math Model Curriculum with Instructional Supports](#) pages 103-107 and [Ohio's High School Geometry Model Curriculum with Instructional Supports](#) pages 89-90.

Discuss with students how not every drawing that has a scale factor is a scale drawing. For example a painting with one point perspective is not a scale drawing although there are scale factors between the figures within the drawing.

DRAWING 3D SHAPES

Although not explicitly in the Common Core or Ohio's Learning Standards for Mathematics, it is important for students to learn how to draw three-dimensional shapes. Exploring the vanishing point is a great way to teach this skill. For additional ideas on how to teach students to draw three-dimensional shapes, see [Ohio's Grade 7 Math Model Curriculum with Instructional Supports](#) pages 107-110.

DILATIONS

Grade 8

A dilation pushes out (or pulls in) every point of the figure from its center of dilation proportionally by the same amount. The results in similar figures. A vanishing point is a great example of a dilation. In Grade 8 the center of dilation is the origin, but in high school it can be any point on the coordinate plane. Through exploration students should discover that dilated figures are similar. If all the corresponding vertices of figures fall on a orthogonal or dilation line ($y = kx$) and the distance between the original vertices and the images' vertices are congruent, then the figures are similar. They should also discover that corresponding line segments are parallel in a dilation. See [Ohio's Grade 8 Math Model Curriculum with Instructional Supports](#) pages 94-98.

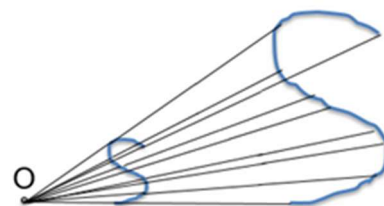
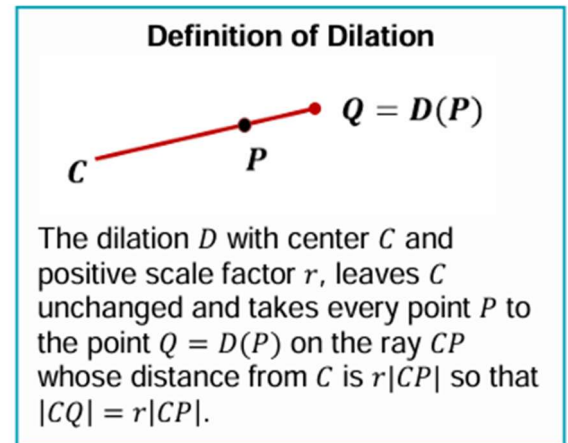


Image taken from [Ohio's Grade 8 Math Model Curriculum with Instructional Supports](#).

An 8th Grade student worksheet on dilations can be found on page 10 of this documents.

High School

In high school Geometry, a dilation is a rule that expands or contracts the plane about a center. The scale factor determines the amount the plane expands (pushes) or contracts (pulls) around the center. It is important to note that in mathematics the entire plane dilates not just the image since the center remains fixed. In high school the center can be any point on the plane (not just the origin) and the scale factor can be positive, negative, or zero. Not only do dilations map lines to lines, rays to rays, and segments to segments, but it maps every line passing through the center of dilation to itself and every line that does not pass through the center to a parallel line. Dilations also preserve angle measure betweenness, and collinearity (distance is not preserved). See [Ohio's High School Geometry Model Curriculum with Instructional Supports](#) pages 79-98 for more information and teaching ideas about dilations.



PART E: CREATING ART USING MATHEMATICS

Work with the fine arts teacher to have students create a drawing/painting using vanishing points. Require them to create 4 pairs of similar figures within the painting/drawing. Then have students write a short paragraph that describes how similar figures on the painting/drawing are dilations. Have them pick a few pairs of similar figures and identify their centers and scale factors.

Here are some websites to help teach one-point perspective:

- [One Point Perspective Drawing: The Ultimate Guide \(studentartguide.com\)](http://studentartguide.com)
- [One Point Perspective City Art Lesson for kids - Leah Newton Art](#)
- [Brain Games- Perspective Cues, Linear Perspective, and Size Contrast \(youtube.com\)](https://www.youtube.com/watch?v=...)

Vanishing Points and Dilations

Teacher Notes

INTRODUCTION

Before introducing One-Point perspective or doing any research about the 0th dimension introduce this question: *How do we double the size of a wiggly curve?* After a short discussion, set the question aside and continue steps A-C.

Note: You may want to review similarity and scale factor. If so, find my lesson plan on Similarity and Vanishing Points on the Word & Numbers math lessons page.

PART A

Go back to your introductory question. Have students discuss how they could use the vanishing point to create a similar curve. Have students try it on their paper. They should have no problem making a similar figure, but the question then becomes how do you know that your new figure is twice the size as the original figure?

Image taken from [Ohio's Grade 8 Math Model Curriculum with Instructional Supports](#).

After much discussion lead students toward assigning an arbitrary point, O, on the plane, and pushing every point on the squiggly line twice as far away from O. Explain to students that this is a dilation. A dilation pushes out (or pulls in) every point of the figure from its center of dilation proportionally by the same amount. This can be easily modeled by pushing in or pulling out an image on an overhead projector or an image drawn on a flashlight. In this case the center of dilation is O, and it can be anywhere on the plane.

If students are stuck, have them try creating a similar rectangle that is twice the size of the original rectangle by using a vanishing point. Then have them measure the orthogonal lines. Ask them what they notice? Have students in the class try a few more examples until they are convinced of their theory.

After their experiments, now have students draw a similar figure of their original curve. Explain to students that they created a dilation and that the center of dilation can be the vanishing point in the picture. Discuss how a dilation always has to have a scale factor and that in 8th grade the vanishing point or center of dilation is the origin and the scale factor has to be positive. Label O (0,0). But point out that in later grades that is not necessarily the case.

- Then give students the challenge question:
 - **Challenge question:** *Can you dilate a point?* Yes, because the plane moves the point also moves, but since a point doesn't have any dimensions the two points are congruent not similar. You can make a larger or smaller dot as a dot's a circle, but not a geometric point.

PART B-D

- Students should discover that the new images are similar to the original figure since the sides are proportional and the angles are preserved.
- Through discussion connect the idea that images created through vanishing points are dilations where the plane expands and contracts. Use a projector or a flashlight to highlight the plane moving.
- Draw attention to the fact that the vanishing point is the center of dilation which in Grade 8 is the origin $(0,0)$. Point out that in high school and beyond the center of dilation can be any point on the plane.
- Students should discover that all circles are similar, but if they are stuck ask them questions that push their thinking.

7. Answer the Reflection Questions:

- What do you notice about the rectangles?

- What's the relationship between the coordinates of rectangle ABCD and rectangle A'B'C'D'?

- What's the relationship between the coordinates of rectangle ABCD and rectangle A''B''C''D''??

- What's the scale factor (or common ratio) between rectangles ABCD and A'B'C'D'?

- What's the scale factor (or common ratio) between rectangles ABCD and A''B''C''D''?

- What conclusions can you make about all 3 rectangles?

- Why can you write a proportion comparing side lengths and widths?
 - Write a proportion describing the lengths and widths of ABCD to A'B'C'D'.
 - Write a proportion describing the lengths and widths of ABCD to A''B''C''D''.

PART C

1. Repeat steps 1-5 in Part A but draw triangles.

2. Find the coordinates of each point on your rectangles.

Triangle	Coordinates of Point A, A' and A''	Coordinates of Point B, B' and B''	Coordinates of Point C, C' and C''
ABC			
A'B'C'			
A''B''C''			

3. Answer the Reflection Questions:

- What do you notice about the triangles?

- What's the relationship between the coordinates of triangle ABC and triangle A'B'C'?

- What's the relationship between the coordinates of triangle ABCD and triangle A''B''C''??

- What's the scale factor (or common ratio) between triangles ABC and A'B'C'?

- What's the scale factor (or common ratio) between triangles ABC and A''B''C''?

- What conclusions can you make about all 3 triangles?

- Why can you write a proportion comparing side lengths and widths?
 - Write a proportion describing the lengths and widths of ABC to A'B'C'.
 - Write a proportion describing the lengths and widths of ABC to A''B''C''.

PART D

4. Repeat steps 1-5 in Part A but draw circles.

5. Find the coordinate of the center and length of the radius of Circle C.

Circle	Coordinates of the center	Length of the radius
C		
C'		
C''		

6. Answer the Reflection Questions:

- How is drawing similar circles different or the same as drawing polygons?

- What's the relationship between the center of circle C and circle C'?

- What's the relationship between the center of circle C and circle C''?

- What's the relationship between the radii of circle C and circle C'?

- What's the relationship between the radii of circle C and circle C''?

- What's the scale factor between circle C and circle C'?

- What's the scale factor between circle C and circle C''?

- Are all circles similar? Explain why or why not?

PART E

Draw a polygon on graph paper. Then using a vanishing point and orthogonal lines draw a similar polygon with a scale factor of 1.5. Make sure to label the vertices of the original figure and its image.

